A Multithreaded Java Framework for Solving Linear Elliptic Partial Differential Equations in 3D

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1 Motivation

Our research in theoretical biophysics [6] recently required us to solve a variant of the Poisson equation, which is a linear elliptic partial differential equation (PDE) in 3D. Ultimately, we want to make our method available as a applet, so we decided to implement a PDE solver in Java.

Another reason for using Java was its promise of easy, portable multithreaded programming, because support for multithreading is built into Java. On a Symmetric Multi-Processor (SMP) this offers the opportunity to painlessly achieve parallel execution, which might be important for the numerically intensive (though certainly not excessive) task of solving a PDE.

The Java framework described here allows the solution of any linear elliptic PDE on a regular cubic domain in 3D. To this end it employs the Full Multigrid (FMG) algorithm [3, 1, 7], which is the most efficient general-purpose algorithm widely used for this problem domain. The framework is easily extensible in every aspect of the PDE algorithm and was designed with interactive Java applets in mind. The framework is implemented in the 1.1 release of Java.

2 The Full Multigrid Algorithm in a Nutshell

This is a brief introduction to the FMG algorithm [3, 1, 7] and the terminology of the FMG framework. The FMG framework deals with solving linear elliptic PDE's on domains in 3D that were discretized by finite differencing; in the following we limit our discussion to this case.

One traditional way of solving a PDE is to iteratively apply a relaxation algorithm starting from an initial guess of the solution. The Multigrid (MG) algorithm, which is at the heart of the FMG method, builds on relaxation algorithms but improves their convergence rate considerably by temporarily working with a coarser grid than the one on which the solution of the PDE is sought.

The MG algorithm looks essentially like this:

1. *Pre-smooth* the approximate solution of the PDE on a grid at a certain level. Grids are organized into levels, where increasing the level by 1 is equivalent to doubling the resolution of the grid. Smoothing is the term for applying a few sweeps of a traditional relaxation algorithm. This leads to an improved solution of the PDE.